

Math 603, Spring 2007, Homework Set 6

Due Friday, March 9th

1. Consider a birth and death chain on $\{0, 1, \dots, d\}$ where d may take the value ∞ . For $x \in \mathcal{I}$, the transition function takes the form

$$P(x, y) = \begin{cases} q_x & : y = x - 1 \\ r_x & : y = x \\ p_x & : y = x + 1 \end{cases}$$

where $q_x + r_x + p_x = 1$, $q_0 = 0$, and $p_d = 0$ if $|\mathcal{I}| < \infty$. Set $\gamma_0 = 1$, and for $0 < i < d$, set

$$\gamma_i = \frac{q_1 \cdots q_i}{p_1 \cdots p_i}$$

Let $a, b \in \mathcal{I}$ with $a < b$. Using the work we discussed in class, show that for all x such that $a < x < b$,

$$P_x(T_b < T_a) = \frac{\sum_{i=a}^{x-1} \gamma_i}{\sum_{i=a}^{b-1} \gamma_i}$$

2. A gambler playing roulette makes a series of one dollar bets. He has respective probabilities $9/19$ and $10/19$ of winning and losing each bet. The gambler decides to quit playing as soon as he either is one dollar ahead or has lost his initial capital of \$1000.

- Find the probability that when he quits playing he will have lost \$1000.
- Find his expected loss.